EFFECT OF COMPRESSIBILITY AND NONISOTHERMICITY ON THE EFFICIENCY OF FILM COOLING IN A TURBULENT BOUNDARY LAYER

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ABSTRACT: Most studies of film cooling deal with the analysis of the thermal shielding efficiency in an incompressible gas flow with constant physical properties. In practice, however, film cooling is used for machine elements exposed to high-speed, high-temperature gas flows. These conditions have received relatively little attention [1–3]. In this paper the influence of these factors on thermal shielding efficiency is analyzed by the method proposed in [4]. It is shown that the effect of compressibility and nonisothermicity on the thermal shielding efficiency is not very great.

From the energy equation of the boundary layer on the thermally insulated part of a plane wall

$$\frac{dR_T^{**}}{dX}+\frac{R_T^{**}}{\Delta T}\frac{d(\Delta T)}{dX}=0 \tag{1}$$

it follows that the thermal shielding efficiency is given by the relation

$$\Theta=\frac{T_0^+-T_w^*}{T_0^+-T_{w0}}=\frac{\delta_{T0}^{**}}{\delta_T^{**}}=\frac{R_{T0}^{**}}{R_T^{**}}$$

$$(\Delta T=T_0^+-T_w,\quad T_0^+=T_0[1+{}^1/_2 r(k-1)M_0^2],\quad k=c_p/c_v),$$

$$R_T^{**}=\frac{\rho_0 w_0\delta_T^{**}}{\mu_{00}},\qquad \delta_T^{**}=\int\limits_0^\infty\frac{\rho w}{\rho_0 w_0}\left(\frac{T_0^+-T^+}{T_0^+-T_w}\right)dy. \tag{2}$$

Here r is the recovery factor, M the Mach number, T_w^* the temperature on the thermally insulated part of the wall in the presence of a film, and δ_{T0}^{**} the energy thickness in the section $x = x_0$.

In [4] it was shown that as $x\to\infty$ on the insulated part of the wall in the presence of film

$$\beta=\frac{\delta_T^{**}}{\delta^{**}}\to\left(\int\limits_0^1\frac{\rho}{\rho_0}\,\omega d\xi\right)\left(\int\limits_0^1\frac{\rho}{\rho_0}\,\omega(1-\omega)\,d\xi\right)^{-1},$$

$$\delta^{**}=\int\limits_0^\infty\frac{\rho w}{\rho_0 w_0}\left(1-\frac{w}{w_0}\right)dy\quad \omega=\frac{w}{w_0},\quad \xi=\frac{y}{\delta}. \tag{3}$$

Here ω is the relative velocity, ξ the relative distance, and δ^{**} the momentum thickness.

In a quasi-isothermal flow with a power-law approximation of the velocity profile to the power n = 1/7

$$\beta\to\beta_{\max}=9. \tag{4}$$

The momentum thickness can be found from a solution of the momentum equation for the boundary layer on a plane wall [5] and with allowance for compressibility and nonisothermicity as $x\to\infty$

$$R^{**}=\left[A(m+1)\int\limits_0^{R_x}\Psi dR_x\right]^{\frac{1}{m+1}}\qquad \left(R^{**}=\frac{\rho_0 w_0\delta^{**}}{\mu_{00}}\right). \tag{5}$$

Here Ψ is a function taking into account the effect of nonisothermicity and compressibility on the friction coefficient, A, m are the coefficient and exponent in the power approximation of the friction law (A = 0.0128, m = 0.25 for a power-law profile with n = 1/7).

The effect of compressibility and nonisothermicity on the friction coefficient at large Reynolds numbers can be taken into account in accordance with the formula [5]

$$\Psi=\left(\frac{C_f}{C_{f_0}}\right)_{R^{**}}=\left[\frac{2\,\mathrm{arc\,tg}\,M\sqrt{0.5r(k-1)}}{(\sqrt{\psi}+1)\,M\sqrt{0.5r(k-1)}}\right]^2\qquad\left(\psi=\frac{T_w}{T_0^+}\right). \tag{6}$$

Here C_f is the friction coefficient for a supersonic gas flow under conditions of nonisothermicity, C_{f_0} is the friction coefficient for subsonic flow over a flat plate with a turbulent boundary layer under quasi-isothermal conditions, and ψ is the temperature factor.

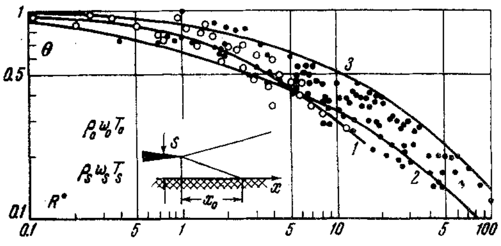

Fig. 1

From Eqs. (2)–(4) we can construct the following interpolation formula for calculating the efficiency of film cooling in a supersonic gas flow under conditions of nonisothermicity:

$$\Theta=\left[1+A(m+1)\beta_{\max}^{m+1}\int\limits_{X_0}^{X}\frac{\Psi R_L}{(R_{T0}^{**})^{m+1}}\,dX\right]^{-\frac{1}{m+1}}. \tag{7}$$

To estimate the effect of compressibility on the coefficient β, in first approximation, in Eq. (3) we assume a similar distribution of the dimensionless stagnation temperatures and velocities over the cross section of the boundary layer. Then

$$\frac{\rho_0}{\rho}=\psi^*-(\psi^*-1)\,\omega^2\qquad\left(\psi^*=1+r\,\frac{k-1}{2}\,M_0^2\right). \tag{8}$$

Here ψ^* is the kinetic temperature factor.

For a power-law approximation of the velocity profile $\omega=\xi^{1/7}$ equation (3) takes the form

$$\beta_{\max}=\left(\int\limits_0^1\frac{\xi^{1/7}d\xi}{\psi^*-(\psi^*-1)\,\xi^{2/7}}\right)\left(\int\limits_0^1\frac{(\xi^{1/7}-\xi^{2/7})\,d\xi}{\psi^*-(\psi^*-1)\,\xi^{2/7}}\right)^{-1}. \tag{9}$$

We present values of the parameter $\beta_{\max}$ calculated from (9) for several values of the Mach number:

M = 0	1	2	3	4	5
β = 9	9.2	9.8	10.4	11.5	12.0.

From this it is clear that up to values M = 3.5 compressibility of the gas does not have an important effect on the coefficient β. Values of the Reynolds number based on the energy thickness in the initial section ($x = x_0$) are found from the following equations.

a) For a plate with an initial heat transfer section, from the solution of the energy equation at T_{w_0} = const

$$R_{T0}^{**}=[A(m+1)\,\Psi R_{x0}]^{\frac{1}{m+1}}. \tag{10}$$

b) For blowing through a tangential slit, by integrating the expression for the energy thickness in the section of the slit we obtain

$$\delta_{T0}^{**}=\frac{\rho_s w_s}{\rho_0 w_0}\,s,\qquad R_{T0}^{**}=\frac{\rho_s w_s s}{\mu_{00}}=R_s\,\frac{\mu_s}{\mu_{00}}. \tag{11}$$

c) For critical blowing through an initial porous section, from the energy equation we have

$$R_{T0}^{**} = \frac{G}{\mu_{00}}, \tag{12}$$

where G is the coolant flow rate per unit width of surface.

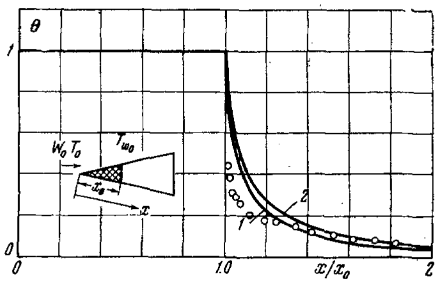

Fig. 2

Thus, all the quantities entering into (7) have been determined with allowance for nonisothermicity and the compressibility of the gas. Since in the general case the function Ψ depends on the unknown temperature T_w^{**}, we shall consider the influence of nonisothermicity in the limiting case, when $\psi = T_{w0}/T_0^+$. In this case Eq. (7) for a power-law profile N = 1/7 can be written in the form

$$\theta \approx \left[1 + 0.016\beta^{1.25}\psi\,\frac{R_{\Delta x}}{R_{T0}^{**1.25}}\right]^{-0.8}. \tag{13}$$

Hence, for the case when coolant gas is blown through a slit at M = 0 and $\psi = 1$, we obtain the known formula [4]

$$\theta \approx \left[1 + 0.24R^\circ\left(\frac{\mu_0}{\mu_s}\right)^{1.25}\right]^{-0.8} \qquad \left(R^\circ = \frac{R_{\Delta x}}{R_s^{1.25}}\right). \tag{14}$$

When M = 0 and $\psi = 0.34$, as in the experiments reported in [2], and at $\beta_{max} = 9$ from (13) we obtain

$$\theta \approx \left[1 + 0.372R^\circ\left(\frac{\mu_0}{\mu_s}\right)^{1.25}\right]^{-0.8} \tag{15}$$

From (14) and (15) it follows that nonisothermicity does not have an appreciable effect on the thermal shielding efficiency.

Figure 1 shows the results of a calculation based on (15) at $\psi = 0.34$ (curve 1), together with calculations based on the equations obtained in [6] for $w_s/w_0 \ll 1$ (curve 2) and for $w_s/w_0 \approx 1$ (curve 3) under quasi-isothermal conditions; for comparison the figure also includes the results of Borodachev's experiments [2] at $\psi = 0.34$ (open circles), and those of Pappel and others [3] at $\psi = 0.6-0.8$ (solid circles).

In [1] the temperature of the insulated wall was measured beyond the heat transfer section on a cone at M = 3.5.

For axisymmetric flow, from the energy equation on the insulated part of the wall

$$\frac{dR_T^{**}}{dX} + R_T^{**}\left[\frac{1}{\Delta T}\frac{d(\Delta T)}{dX} + \frac{1}{R}\frac{dR}{dX}\right] = 0 \tag{16}$$

it follows that

$$\theta = \frac{T_0^+ - T_w^*}{T_0^+ - T_{w0}} = \frac{D_0}{D}\frac{R_{T0}^{**}}{R_T^{**}}. \tag{17}$$

Then from Eqs. (10), (13), (17) the thermal shielding efficiency on the cone is given by

$$\theta_k \approx \frac{x_0}{x}\left[1 + 15.5\left(\frac{x - x_0}{x_0}\right)\right]^{-0.8}$$
$$(M = 0,\ \beta = 9), \tag{18}$$

$$\theta_k \approx \frac{x_0}{x}\left[1 + 19.7\left(\frac{x - x_0}{x_0}\right)\right]^{-0.8}$$
$$(M = 3.5,\ \beta_{max} = 10.9). \tag{19}$$

Figure 2 gives the results of calculations based on (19)—curve 1, and on (18)—curve 2; the data of experiments [1]—open circles—are given for comparison. In this case the effect of compressibility on the thermal shielding efficiency was expressed in (13) only through the coefficient β_{max}. However, when shielding takes the form of blowing coolant into the boundary layer and the quantity R_{T0}^{**} does not contain the parameter Ψ (see Eqs. (11) and (12)), then, in accordance with (13), the compressibility of the gas exerts an influence on the efficiency θ through the functions β and Ψ. With increase in Mach number the changes in the quantities β_{max} and Ψ to some extent compensate one another.

Thus, it may be assumed that nonisothermicity and compressibility do not have much effect on the insulating efficiency in film cooling, and in the first approximation many practical calculations can be based on the equations obtained for quasi-isothermal conditions.

REFERENCES

1. F. H. Durgin, "An 'insulating' boundary layer experiment," J. Aerospys. Sci., vol. 26, no. 6, 1959.

2. V. N. Borodachev, Theoretical and Experimental Investigation of Air-Film Cooling of a Flat Plate [in Russian], Oborongiz, 1956.

3. I. L. Stollery and A. A. El-Ehwany, "A note on the use of a boundary layer model for correlating film-cooling data," Internat. J. Heat and Mass Transfer, vol. 8, no. 1, 1965.

4. S. S. Kutateladze and A. I. Leont'ev, "Thermal shielding in a turbulent gas boundary layer," Teplofiz. vysokikh temperatur, no. 2, 281–290, 1963.

5. S. S. Kutateladze and A. I. Leont'ev, Turbulent Boundary Layer of a Compressible Gas [in Russian], Izd. SO AN SSSR, 1962.

6. E. P. Volchkov and V. Ya. Levchenko, "Efficiency of a gas curtain in a turbulent boundary layer," PMTF [Journal of Applied Mechanics and Technical Physics], no. 5, 1965.

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